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Energies, Luminosities, and Timescales:

Here we estimate certain critical numbers characterizing the high-energy emission by compact objects accreting from their environment. These considerations will be important for building high-energy telescopes and effectively using them for signals reaching Earth from distant sources.

The principal source of power in diverse sources, like X-ray binaries and active galactic nuclei (AGN), is the release of gravitational potential energy.

Consider a mass m falling onta an object of bigger mass M and radius R. The released energy is:

$$\Delta E_{acc} = \frac{GM}{R}$$

To put things in perspective, lets compare ΔE_{acc} with ΔE_{nuc}

released in the pp-chain (hydrogen burning) for various objects. We have:

$\Delta E_{nuc} (H \to He) \simeq 6 \times 10^{18}$ erg g^{-1}

For a neutron star $M \sim 1 M_{\odot}$ and $R \sim 10$ km, which results in:

$\Delta E_{acc} (ns) \simeq 10^{20}$ erg g^{-1} $\sim 20\, \Delta E_{nuc}$

For a white dwarf $M \sim 1 M_{\odot}$ and $R \sim 10{,}000$ km, resulting in:

$\Delta E_{acc} (wd) \simeq 10^{17}$ erg g^{-1} $\sim \frac{1}{50} \Delta E_{nuc}$

For a black hole, we should use the Schwarzschild radius

$R_s = \frac{2GM}{c^2}$, which leads to:

$\Delta E_{acc} (bh) \simeq 5 \times 10^{20}$ erg g^{-1} $\sim 100\, \Delta E_{nuc}$

Two additional questions arise regarding energy released by gravity:

1) What is the total luminosity associated with the process?

2) How quickly can the emission region vary?

To answer the first question, lets consider a plasma of ionized hydrogen that is in hydrostatic equilibrium. In this case, the inward pull of gravity is balanced with the outward push by radiation.

Photons diffusing outward interact with the electrons and protons. The cross-section for scattering is $\propto \frac{1}{m^2}$, and hence it is much larger for photon scattering off electrons. If we consider Thomson scattering, the total scattering cross-section is given by that for photon-electron scattering σ_T.

Now consider the gravitational force on an electron-proton pair. It follows:

$$f_{grav} = \frac{-GM(m_p + m_e)}{r^2} \simeq -\frac{GMm_p}{r^2}$$

This must be balanced by the force from photons. The momentum carried by radiation with energy flux F is;

$$\Pi = \frac{F}{c}$$

For Thomson scattering cross-section σ_T, the force from radiation is therefore given by:

$$f_{rad} = \frac{F}{c}\sigma_T$$

The two forces cancel each other when:

$$\frac{F}{c}\sigma_T = \frac{GMm_p}{r^2}$$

The flux F at a distance r is related to the luminosity L according to:

$$F = \frac{L}{4\pi r^2}$$

Thus, in equilibrium we have:

$$\frac{\sigma_T L}{4\pi c} = GMm_p$$

This yields a critical luminosity, called the Eddington limit L_{edd}, for which the equilibrium holds:

$$L_{edd} \equiv \frac{4\pi c G M m_p}{\sigma_T}$$

Numerically, we have:

$L_{edd} \simeq 1.3 \times 10^{38} \left(\frac{M}{M_{\odot}}\right) \text{ erg s}^{-1}$

For a neutron star with $M = 1.4\, M_{\odot}$:

$L_{edd}(ns) \sim 10^{4}\, L_{\odot}$

For an AGN black hole with $M = 10^{7}\, M_{\odot}$, we have:

$L_{edd}(AGN) \sim 7 \times 10^{11}\, L_{\odot}$

If $L > L_{edd}$, the source will expel nearby matter and quench the process of accretion.

The effective temperature T_{eff} of a blackbody with total power L_{edd} is:

$$T_{eff} = \left(\frac{G M m_p}{R^2 \sigma_T \sigma_B}\right)^{\frac{1}{4}}$$

For a neutron star $T_{eff} \sim 2 \times 10^{7}$ K, and the charactristic photon energy is ~ 1.6 keV (X-rays).

For a white dwarf, $T_{eff} \sim 6 \times 10^5$ K, which results in a characteristic energy ~ 50 eV (UV).

For a black hole the situation is more complicated because the gravitational energy is released over an extended region of space (due to the absence of a hard surface).

The photon number flux on Earth from a source at the center of galaxy is:

$$F_{ph} \doteqdot \frac{L_{edd}}{4\pi (8.5 \text{ kpc})^2 \langle \epsilon \rangle}$$

$\langle \epsilon \rangle$ is the average photon energy. For an X-rays, $\langle \epsilon \rangle \sim$ 1.6 keV, which results in:

$F_{ph} \sim 10 \text{ cm}^{-2} \text{ s}^{-1}$

The Chandra observatory achieves a sensitivity of about $2 \times 10^{-6} \text{ cm}^{-2} \text{ s}^{-1}$, which allows it to discover very faint objects.

Now lets turn to the second question in above. For gravity, the variability timescale t_{var} is influenced by the dynamical timescale t_{dyn}. This provides an upper limit on t_{var}. For an element of mass m falling under gravity, t_{dyn} can be estimated as follows. The time that it takes for the mass to fall a distance R (i.e, radius of the star) is:

$$t_{dyn} = \left(\frac{2R}{a}\right)^{\frac{1}{2}} = \left(\frac{2R^3}{GM}\right)^{\frac{1}{2}}$$

Here we use $a = \frac{GM}{R^2}$ for the gravitational acceleration.

For a neutron star:

$$t_{dyn}(ns) \sim 10^{-4}\,s$$

For a white dwarf:

$$t_{dyn}(wd) \sim 3\,s$$

The Doppler shift and light-travel time effects can seriously modify t_{var} inferred by a distant observer.

It is important to note that an instrument should accumulate reasonable number of photons to measure the incoming flux from typical high-energy sources in several seconds, if it is going to detect changes in the emitter's configuration. In the case of a neutron star, integration time is much shorter (a fraction of a millisecond).